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Robust Stabilization of the Space Station in the Presence of Inertia Matrix Uncertainty

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The robust stabilization problem of the space station in the face of inertia matrix uncertainty is formulated as a robust H_∞ full-state feedback control problem with direct coupling between the controlled output and disturbance input. The control design objective is to yield the largest stable hypercube in uncertain parameter space, while satisfying the nominal performance requirements. The significance of employing an uncertain plant model with direct coupling between the controlled output and disturbance input is demonstrated.

I. Introduction

IN recent years there has been a growing interest in robust stabilization of the space station using various advanced control design techniques. Consequently, substantial contributions have already been made to the space station control problem, as evidenced in Refs. 1–9. The space station is in fact a flexible multibody vehicle with time-varying inertias; however, it can be considered as a single rigid body for the practical design of a low-bandwidth, integrated attitude/momentum controller. All physical states of the vehicle will become available from a strapdown inertial reference system of the vehicle. For this reason, a full-state feedback control design problem has been extensively studied for the space station in the past.[1–9] It is also emphasized that this paper is not concerned with the problem of controlling the space station in the presence of significant changes of inertias during the assembly sequence and Space Shuttle docking.[9] This paper is mainly concerned with the problem of designing a constant-gain controller that may yield the largest stable hypercube in uncertain parameter space, subject to the nominal performance requirements.

In Ref. 5 a robust H_∞ control design methodology, which incorporates the concepts of fictitious inputs and outputs and linearized directional variations of nonlinearly related uncertain parameters, was applied to the space station control problem. The technique has yielded a remarkable result in stability robustness with respect to the moments-of-inertia variation in one of the structured uncertainty directions. However, such a technique sometimes fails to provide the largest stable hypercube in uncertain parameter space. The technique may maximize a parameter robustness measure along a "known" uncertainty direction, but the largest stable hypercube may actually touch the instability boundary at other points in the uncertain parameter space.

As first shown in Ref. 10, structured uncertainty modeling of a certain class of dynamical systems with uncertain inertia matrices results in an uncertain plant model with $D_{11} \neq 0$, where D_{11} is a matrix that relates the disturbance input and controlled output.[5,10–13] In this paper, we present a robust H_∞ full-state feedback control synthesis method for such an uncertain system with $D_{11} \neq 0$. Although a solution to the H_∞ full-state feedback control problem can be found in Refs. 14 and 15, the method presented in this paper utilizes proper scaling and unimodular transformation[13] so that the Glover-Doyle algorithm[11] can be directly employed to solve the H_∞ full-state feedback control problem with the nonzero D_{11} term. The method is then applied to the robust stabilization problem of the space station in the face of inertia matrix uncertainty.

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II. Robust H_∞ Full-State Feedback Control

Structured Uncertainty Modeling

Consider a dynamical system described by

$$E\dot{x} = Fx + G_d d + G_u u \tag{1}$$

where x, d, and u are the state, disturbance input, and control input vectors, respectively; G_d is the disturbance input distribution matrix; G_u is the control input distribution matrix; and the matrices E and F are subject to structured parameter variations.

Suppose that there are ℓ independent, uncertain parameter variables δ_i, and assume that the perturbed matrices E and F in Eq. (1) can be linearly decomposed as follows:

$$E = E_0 + \Delta E \tag{2a}$$

$$F = F_0 + \Delta F \tag{2b}$$

where E_0 and F_0 are the nominal matrices, and ΔE and ΔF are the perturbation matrices defined as

$$\Delta E = \sum_{i=1}^{\ell} \Delta E_i \delta_i = \sum_{i=1}^{\ell} M_E^{(i)} \delta_i I_{\kappa_i} N_E^{(i)} = M_E \mathcal{E}_E N_E \tag{3a}$$

$$\Delta F = \sum_{i=1}^{\ell} \Delta F_i \delta_i = \sum_{i=1}^{\ell} M_F^{(i)} \delta_i I_{\nu_i} N_F^{(i)} = M_F \mathcal{E}_F N_F \tag{3b}$$

where κ_i is the rank of ΔE_i, ν_i is the rank of ΔF_i, and $\mathcal{E}_E$ and $\mathcal{E}_F$ are diagonal matrices with δ_i as their diagonal elements. If $\kappa_i = \nu_i = 1$ for $i = 1, \ldots, \ell$ (i.e., a special case of rank-one dependency), $M_E^{(i)}$ and $M_F^{(i)}$ become column vectors and $N_E^{(i)}$ and $N_F^{(i)}$ become row vectors. In this case, there are no repeated elements δ_i in $\mathcal{E}_E$ and $\mathcal{E}_F$.

Let

$$\mathcal{E} \triangleq \text{diag}\{\mathcal{E}_E,\ \mathcal{E}_F\} \tag{4a}$$

$$\tilde{z} \triangleq \begin{bmatrix} \tilde{z}_E \\ \tilde{z}_F \end{bmatrix} = \begin{bmatrix} N_E \dot{x} \\ N_F x \end{bmatrix} \tag{4b}$$

$$\tilde{d} \triangleq -\mathcal{E}\tilde{z} \tag{4c}$$

where $\tilde{d}$ is called the fictitious disturbance input, $\tilde{z}$ the fictitious output, and $\mathcal{E}$ the gain matrix of fictitious internal feedback loop, which is caused by uncertainty in the matrices E and F. Then substituting Eqs. (2) into Eq. (1), we obtain

$$E_0\dot{x} = F_0 x + G_{\tilde{d}}\tilde{d} + G_d d + G_u u \tag{5}$$

where $G_{\tilde{d}}$, the fictitious disturbance distribution matrix, is defined as

$$G_{\tilde{d}} = [M_E \quad -M_F]$$

Defining the controlled output vector as

$$z = \begin{bmatrix} C_{11} \\ 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ I \end{bmatrix} u$$

and introducing new variables

$$\hat{d} \triangleq \begin{bmatrix} \tilde{d} \\ d \end{bmatrix}, \qquad \hat{z} \triangleq \begin{bmatrix} \tilde{z} \\ z \end{bmatrix}$$

we obtain a modified state-space representation of the system as follows:

$$\dot{x} = Ax + B_1\hat{d} + B_2 u \tag{6a}$$

$$\hat{z} = C_1 x + D_{11}\hat{d} + D_{12}u \tag{6b}$$

where

$$A = E_0^{-1}F_0, \qquad B_1 = E_0^{-1}[G_{\tilde{d}} \quad G_d], \qquad B_2 = E_0^{-1}G_u$$

$$C_1 = \begin{bmatrix} N_E E_0^{-1}F_0 \\ N_F \\ C_{11} \\ 0 \end{bmatrix}, \qquad D_{11} = N_E E_0^{-1}\begin{bmatrix} G_{\tilde{d}} & G_d \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$D_{12} = \begin{bmatrix} N_E E_0^{-1}G_u \\ 0 \\ 0 \\ I \end{bmatrix}$$

Note that $D_{11} = 0$ if there is no uncertainty in E.

H_∞ Full-State Feedback for $D_{11} = 0$

Let $T_{zd}(s)$ be the closed-loop transfer function matrix from d to z, but with the internal uncertainty loop closed. Also define $T_{\hat{z}\hat{d}}$ to be the closed-loop transfer function matrix, but with the internal uncertainty loop broken, as follows:

$$\hat{z} = \begin{bmatrix} \tilde{z} \\ z \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}\begin{bmatrix} \tilde{d} \\ d \end{bmatrix} = T_{\hat{z}\hat{d}}\hat{d} \tag{7}$$

Furthermore, $T_{zd}(s)$ and $T_{\hat{z}\hat{d}}$ are related as

$$T_{zd} = T_{22} - T_{21}\mathcal{E}(I + T_{11}\mathcal{E})^{-1}T_{12} \tag{8}$$

We then have the following two theorems that provide sufficient conditions for stability/performance robustness.[5]

Theorem 1 (Stability Robustness): If $\|T_{11}(s)\|_\infty < \gamma$, then $T_{zd}(s, \alpha\mathcal{E})$ $\forall\alpha \in [0, 1]$ is stable for $\|\mathcal{E}\| \le \gamma^{-1}$.

Theorem 2 (Performance Robustness): If $\|T_{\hat{z}\hat{d}}\|_\infty < \gamma$, then $T_{zd}(s, \alpha\mathcal{E})$ $\forall\alpha \in [0, 1]$ is stable and $\|T_{zd}(s, \alpha\mathcal{E})\|_\infty < \gamma$ $\forall\alpha \in [0, 1]$ with $\|\mathcal{E}\| \le \gamma^{-1}$.

The following theorem[11,12] provides a robust H_∞-suboptimal controller that satisfies the condition in Theorem 2.

Theorem 3 (H_∞ Full-State Feedback): Assume the following:

a) (A, B_2) is stabilizable.

b) $D_{12} = [0 \quad I]^T$, and $D_{11} = 0$.

c) $\begin{bmatrix} A - j\omega I & B_2 \\ C_1 & D_{12} \end{bmatrix}$ has full column rank for all ω.

Then there exists an internally stabilizing controller such that $\|T_{\hat{z}\hat{d}}\|_\infty < \gamma$, if and only if the following Riccati equation

$$\left(A - B_2D_{12}^TC_1\right)^T X + X\left(A - B_2D_{12}^TC_1\right) - X\left(B_2B_2^T - \frac{1}{\gamma^2}B_1B_1^T\right)X + C_1^T\left(I - D_{12}D_{12}^T\right)C_1 = 0 \tag{9}$$

has a solution $X \ge 0$. The state feedback gain matrix K of $u = -Kx$, which satisfies $\|T_{zd}\|_\infty < \gamma$, is then obtained as

$$K = B_2^TX + D_{12}^TC_1 \tag{10}$$

H_∞ Full-State Feedback for $D_{11} \neq 0$

Consider a linear, time-invariant system described by Eqs. (6), where u is the m_2-dimensional control input vector and $\hat{z}$ is the p_1-dimensional output vector. Although a full-state feedback gain matrix can be computed using Theorem 3, it is not always possible to have a system description that meets assumption b: $D_{12} = [0 \quad I]^T$ and $D_{11} = 0$. Although a general solution to the H_∞ full-state feedback control problem can be found in Refs. 14 and 15, we present here a computational procedure, which is based on proper scaling and unimodular transformation of Ref. 13, for synthesizing a full-state feedback gain matrix for a system description with $D_{12} \neq [0 \quad I]^T$, $D_{11} \neq 0$, and $D_{11}^TD_{11} < \gamma^2 I$. Such a procedure is briefly outlined as follows.

Step 1: Scale D_{12}.

First perform the singular value decomposition of D_{12} as

$$D_{12} = [\underbrace{U_1}_{m_2} \quad \underbrace{U_2}_{(p_1-m_2)}]\begin{bmatrix} \Sigma \\ 0 \end{bmatrix}V^T \tag{11}$$

Then define new scaled input $u^{(1)}$ and output $\hat{z}^{(1)}$ as

$$u = S_u^{(1)}u^{(1)} \triangleq V\Sigma^{-1}u^{(1)} \tag{12a}$$

$$\hat{z}^{(1)} = S_{\hat{z}}^{(1)}\hat{z} \triangleq \begin{bmatrix} U_2^T \\ U_1^T \end{bmatrix}\hat{z} \tag{12b}$$

and let $\hat{d}^{(1)} = \hat{d}$. Substituting Eqs. (12) into Eqs. (6), we obtain the following state-space equations:

$$\dot{x} = A^{(1)}x + B_1^{(1)}\hat{d}^{(1)} + B_2^{(1)}u^{(1)}$$

$$\hat{z}^{(1)} = C_1^{(1)}x + D_{11}^{(1)}\hat{d}^{(1)} + D_{12}^{(1)}u^{(1)}$$

where

$$A^{(1)} = A, \qquad B_1^{(1)} = B_1, \qquad B_2^{(1)} = B_2S_u^{(1)}$$

$$C_1^{(1)} = S_{\hat{z}}^{(1)}C_1, \qquad D_{11}^{(1)} = S_{\hat{z}}^{(1)}D_{11}, \qquad D_{12}^{(1)} = [0 \quad I]^T$$

Step 2: Scale D_{11}. Using the unimodular transformation[13]:

$$\begin{bmatrix} \hat{z}^{(1)} \\ \hat{d}^{(1)} \end{bmatrix} = \begin{bmatrix} X_1^{-\frac{1}{2}} & X_1^{-\frac{1}{2}}D_{11}^{(1)} \\ \left\{D_{11}^{(1)}\right\}^T X_1^{-\frac{1}{2}} & X_2^{-\frac{1}{2}} \end{bmatrix}\begin{bmatrix} \hat{z}^{(2)} \\ \hat{d}^{(2)} \end{bmatrix} \tag{13}$$

where

$$X_1 = I - D_{11}^{(1)}\left\{D_{11}^{(1)}\right\}^T$$

$$X_2 = I - \left\{D_{11}^{(1)}\right\}^T D_{11}^{(1)}$$

we obtain the following state-space equations:

$$\dot{x} = A^{(2)}x + B_1^{(2)}\hat{d}^{(2)} + B_2^{(2)}u^{(2)} \tag{14a}$$

$$\hat{z}^{(2)} = C_1^{(2)}x + D_{11}^{(2)}\hat{d}^{(2)} + D_{12}^{(2)}u^{(2)} \tag{14b}$$

where $u^{(2)} = u^{(1)}$ and

$$A^{(2)} = A^{(1)} + B_1^{(1)}\{D_{11}^{(1)}\}^T X_1^{-1} C_1^{(1)}$$

$$B_1^{(2)} = B_1^{(1)} X_2^{-\frac{1}{2}}$$

$$B_2^{(2)} = B_2^{(1)} + B_1^{(1)}\{D_{11}^{(1)}\}^T X_1^{-1} D_{12}^{(1)}$$

$$C_1^{(2)} = X_1^{-\frac{1}{2}} C_1^{(1)}$$

$$D_{11}^{(2)} = 0$$

$$D_{12}^{(2)} = X_1^{-\frac{1}{2}} D_{12}^{(1)}$$

Step 3: Repeat step 1 to rescale $D_{12}^{(2)}$.

Perform the singular value decomposition of $D_{12}^{(2)}$ as follows:

$$D_{12}^{(2)} = [\underbrace{U_1}_{m_2} \quad \underbrace{U_2}_{(p_1-m_2)}]\begin{bmatrix}\Sigma \\ 0\end{bmatrix} V^T \tag{15}$$

where the matrices U_1, U_2, Σ, and V are different from those in Eq. (11) for D_{12}. Then define new scaled input $u^{(3)}$ and output $\hat{z}^{(3)}$ as follows:

$$u^{(2)} = S_u^{(3)} u^{(3)} \triangleq V\Sigma^{-1} u^{(3)} \tag{16a}$$

$$\hat{z}^{(3)} = S_{\hat{z}}^{(3)} \hat{z}^{(2)} \triangleq \begin{bmatrix} U_2^T \\ U_1^T \end{bmatrix} \hat{z}^{(2)} \tag{16b}$$

Substituting Eqs. (16) into Eqs. (14), we obtain the following state-space equations:

$$\dot{x} = A^{(3)} x + B_1^{(3)} \hat{d}^{(3)} + B_2^{(3)} u^{(3)} \tag{17a}$$

$$\hat{z}^{(3)} = C_1^{(3)} x + D_{11}^{(3)} \hat{d}^{(3)} + D_{12}^{(3)} u^{(3)} \tag{17b}$$

where $\hat{d}^{(3)} = \hat{d}^{(2)}$ and

$$A^{(3)} = A^{(2)}, \qquad B_1^{(3)} = B_1^{(2)}, \qquad B_2^{(3)} = B_2^{(2)} S_u^{(3)}$$

$$C_1^{(3)} = S_{\hat{z}}^{(3)} C_1^{(2)}, \qquad D_{11}^{(3)} = S_{\hat{z}}^{(3)} D_{11}^{(2)} = 0, \qquad D_{12}^{(3)} = [0 \quad I]^T$$

Step 4: Determine K.

After computing the feedback gain matrix $K^{(3)}$ for the system described by Eqs. (17) using Theorem 3, we obtain the actual gain matrix K as follows:

$$K = S_u^{(1)} S_u^{(3)} K^{(3)} \tag{18}$$

since $u = S_u^{(1)} u^{(1)} = S_u^{(1)} S_u^{(3)} u^{(3)} = S_u^{(1)} S_u^{(3)} K^{(3)} x$.

It is noted that a general solution to the H_∞ full-state feedback control problem can be found in Refs. 14 and 15.

III. Linearized Model of the Space Station

The space station in a circular orbit is expected to maintain its local-vertical and local-horizontal (LVLH) orientation using control moment gyros (CMGs). For small attitude deviations from the desired LVLH orientation, the linearized equations of motion about the principal axes can be found as follows[1]:

Space station dynamics:

$$\begin{aligned} J_1\dot{\omega}_1 + n(J_2 - J_3)\omega_3 + 3n^2(J_2 - J_3)\theta_1 &= -\tau_1 + d_1 \\ J_2\dot{\omega}_2 + 3n^2(J_1 - J_3)\theta_2 &= -\tau_2 + d_2 \\ J_3\dot{\omega}_3 - n(J_2 - J_1)\omega_1 &= -\tau_3 + d_3 \end{aligned} \tag{19}$$

Attitude kinematics:

$$\begin{aligned} \dot{\theta}_1 - n\theta_3 &= \omega_1 \\ \dot{\theta}_2 - n &= \omega_2 \\ \dot{\theta}_3 + n\theta_1 &= \omega_3 \end{aligned} \tag{20}$$

CMG momentum:

$$\begin{aligned} \dot{h}_1 - nh_3 &= \tau_1 \\ \dot{h}_2 &= \tau_2 \\ \dot{h}_3 + nh_1 &= \tau_3 \end{aligned} \tag{21}$$

where θ_1, θ_2, and θ_3 are the small roll, pitch, and yaw angles of the body axes with respect to the LVLH reference frame that rotates at the orbital rate of $n = 0.0011$ rad/s; ω_1, ω_2, and ω_3 are the body-axis components of the absolute angular velocity of the station; h_1, h_2, and h_3 are the body-axis components of the CMG momentum; τ_1, τ_2, and τ_3 are the body-axis components of the control torque; d_1, d_2, and d_3 are the body-axis components of the external disturbance torque; and J_1, J_2, and J_3 are the principal moments of inertia.

In this paper, we consider a particular configuration with the following nominal values of inertias in units of slug-ft^2 (Refs. 1–6): $J_1 = 50.28 \times 10^6$, $J_2 = 10.80 \times 10^6$, and $J_3 = 58.57 \times 10^6$. It is assumed that for this particular configuration the external aerodynamic disturbance torques in units of ft-lb are modeled as

$$\begin{aligned} d_1(t) &= 1 + \sin(nt) + 0.5\sin(2nt) \\ d_2(t) &= 4 + 2\sin(nt) + 0.5\sin(2nt) \\ d_3(t) &= 1 + \sin(nt) + 0.5\sin(2nt) \end{aligned} \tag{22}$$

However, the magnitudes and phases of cyclic components are assumed to be unknown for control design.

To minimize the attitude or CMG momentum oscillations caused by cyclic terms in the external disturbances, the periodic-disturbance rejection filters are also considered in this paper. Such filters for the disturbance accommodating control of h_1, θ_2, and θ_3 can be represented as [1,2]

$$\ddot{\alpha}_i + (n)^2\alpha_i = y_i \tag{23a}$$

$$\ddot{\beta}_i + (2n)^2\beta_i = y_i \tag{23b}$$

where $y_1 = h_1$, $y_2 = \theta_2$, and $y_3 = \theta_3$.

The pitch control logic is expressed as

$$\tau_2 = K_{22}\,\bar{x}_2 \tag{24}$$

where K_{22} is a 1×8 gain matrix and $\bar{x}_2$ is the pitch-axis state vector defined as

$$\bar{x}_2 \triangleq \begin{bmatrix} \theta_2 & \dot{\theta}_2 & h_2 & \int h_2 & \alpha_2 & \dot{\alpha}_2 & \beta_2 & \dot{\beta}_2 \end{bmatrix}^T$$

The CMG momentum and its integral are included as state variables to prevent undesirable CMG momentum buildup.

Similarly, the roll/yaw control logic can be expressed as

$$\begin{bmatrix} \tau_1 \\ \tau_3 \end{bmatrix} = \begin{bmatrix} K_{11} & K_{13} \\ K_{31} & K_{33} \end{bmatrix} \begin{bmatrix} \bar{x}_1 \\ \bar{x}_3 \end{bmatrix} \tag{25}$$

where the various K_{ij} are 1×8 gain matrices and

$$\bar{x}_1 \triangleq \begin{bmatrix} \theta_1 & \omega_1 & h_1 & \int h_1 & \alpha_1 & \dot{\alpha}_1 & \beta_1 & \dot{\beta}_1 \end{bmatrix}^T$$

$$\bar{x}_3 \triangleq \begin{bmatrix} \theta_3 & \omega_3 & h_3 & \int h_3 & \alpha_3 & \dot{\alpha}_3 & \beta_3 & \dot{\beta}_3 \end{bmatrix}^T$$

In this paper, we introduce a nondimensionalized or scaled math model of the space station to avoid a numerical sensitivity problem inherent to solving Riccati equations of the space station, as follows:

Space station dynamics:

$$\begin{aligned}\frac{J_1}{J_1^0}\dot\omega_1 + \frac{J_2-J_3}{J_1^0}\omega_3 + 3\frac{J_2-J_3}{J_1^0}\theta_1 &= -\tau_1 + d_1\\ \frac{J_2}{J_2^0}\dot\omega_2 + 3\frac{J_1-J_3}{J_2^0}\theta_2 &= -\tau_2 + d_2\\ \frac{J_3}{J_3^0}\dot\omega_3 - \frac{J_2-J_1}{J_3^0}\omega_1 &= -\tau_3 + d_3\end{aligned} \tag{26}$$

Attitude kinematics:

$$\begin{aligned}\dot\theta_1 - \theta_3 &= \omega_1\\ \dot\theta_2 - 1 &= \omega_2\\ \dot\theta_3 + \theta_1 &= \omega_3\end{aligned} \tag{27}$$

CMG momentum:

$$\begin{aligned}\dot h_1 - \frac{J_3^0}{J_1^0}h_3 &= \tau_1\\ \dot h_2 &= \tau_2\\ \dot h_3 + \frac{J_1^0}{J_3^0}h_1 &= \tau_3\end{aligned} \tag{28}$$

Periodic-disturbance rejection filters:

$$\begin{aligned}\ddot\alpha_1 + \alpha_1 &= h_1\\ \ddot\beta_1 + 4\beta_1 &= h_1\\ \ddot\alpha_2 + \alpha_2 &= \theta_2\\ \ddot\beta_2 + 4\beta_2 &= \theta_2\\ \ddot\alpha_3 + \alpha_3 &= \theta_3\\ \ddot\beta_3 + 4\beta_3 &= \theta_3\end{aligned} \tag{29}$$

where time is in units of n; and J_1^0, J_2^0, and J_3^0 are the nominal roll, pitch, and yaw principal moments of inertia, respectively. All variables in the preceding equations (26–29) are properly scaled such that the state feedback gain matrices K_{ij}, defined in Eqs. (24) and (25) for the original model, and the state feedback gain matrices K'_{ij} of the nondimensionalized model are related as

$$\begin{aligned}K_{22} &= K'_{22}S_2\\ [K_{11}\quad K_{13}] &= [K'_{11}\quad K'_{13}]S_1\\ [K_{31}\quad K_{33}] &= [K'_{31}\quad K'_{33}]S_3\end{aligned} \tag{30}$$

where

$$S_1 = J_1^0\cdot\text{diag}\left\{n^2, n, \frac{n}{J_1^0}, \frac{n^2}{J_1^0}, \frac{n^3}{J_1^0}, \frac{n^2}{J_1^0}, \frac{n^3}{J_1^0}, \frac{n^2}{J_1^0}, n^2, n, \frac{n}{J_3^0}, \frac{n^2}{J_3^0}, n^4, n^3, n^4, n^3\right\}$$

$$S_2 = J_2^0\cdot\text{diag}\left\{n^2, n, \frac{n}{J_2^0}, \frac{n^2}{J_2^0}, n^4, n^3, n^4, n^3\right\}$$

$$S_3 = J_3^0\cdot\text{diag}\left\{n^2, n, \frac{n}{J_1^0}, \frac{n^2}{J_1^0}, \frac{n^3}{J_1^0}, \frac{n^2}{J_1^0}, \frac{n^3}{J_1^0}, \frac{n^2}{J_1^0}, n^2, n, \frac{n}{J_3^0}, \frac{n^2}{J_3^0}, n^4, n^3, n^4, n^3\right\}$$

IV. Pitch Control Design

The nondimensionalized model for pitch-axis control design is written as

$$\begin{bmatrix}1 & 0\\ 0 & J_2/J_2^0\end{bmatrix}\begin{bmatrix}\dot\theta_2\\ \ddot\theta_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -3(J_1-J_3)/J_2^0 & 0\end{bmatrix}\begin{bmatrix}\theta_2\\ \dot\theta_2\end{bmatrix} + \begin{bmatrix}0\\ -1\end{bmatrix}\tau_2 \tag{31}$$

The external aerodynamic disturbance $d_2(t)$ is not included here since it is accommodated by the disturbance rejection filter.

The perturbed moments of inertia are modeled as

$$J_i = J_i^0(1+\delta_i), \qquad i = 1, 2, 3 \tag{32}$$

where δ_i represents the percentage variation from the nominal values of each parameter.

Substituting Eq. (32) into Eq. (31) and comparing the result with Eq. (1), we obtain

$$E = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} + \begin{bmatrix}0 & 0\\ 0 & \delta_2\end{bmatrix} = E_0 + \Delta E$$

$$F = \begin{bmatrix}0 & 1\\ -3k_2 & 0\end{bmatrix} + \begin{bmatrix}0 & 0\\ -3k_{12}\delta_1 + 3k_{32}\delta_3 & 0\end{bmatrix} = F_0 + \Delta F$$

where

$$k_2 = \frac{J_1^0 - J_3^0}{J_2^0}, \qquad k_{ij} = \frac{J_i^0}{J_j^0}$$

The perturbation matrices ΔE and ΔF can be decomposed as

$$\Delta E = M_E\mathcal{E}_E N_E = \begin{bmatrix}0\\ 1\end{bmatrix}\delta_2[0\quad 1]$$

$$\begin{aligned}\Delta F &= M_F\mathcal{E}_F N_F\\ &= \begin{bmatrix}0 & 0\\ -3k_{12} & 3k_{32}\end{bmatrix}\begin{bmatrix}\delta_1 & 0\\ 0 & \delta_3\end{bmatrix}\begin{bmatrix}1 & 0\\ 1 & 0\end{bmatrix}\end{aligned}$$

Defining the following new variables

$$\tilde z \triangleq \begin{bmatrix}\tilde z_1\\ \tilde z_2\\ \tilde z_3\end{bmatrix} = \begin{bmatrix}\theta_2\\ \ddot\theta_2\\ \theta_2\end{bmatrix}$$

$$\tilde d \triangleq \begin{bmatrix}\tilde d_1\\ \tilde d_2\\ \tilde d_3\end{bmatrix} = -\mathcal{E}\tilde z$$

where

$$\mathcal{E} = \text{diag}\{\delta_1, \delta_2, \delta_3\}$$

we obtain a state-space model of the pitch-axis dynamics, augmented by the disturbance filter model and the fictitious internal feedback loop, as follows:

$$\begin{aligned}\dot x &= Ax + B_1\hat d + B_2u\\ \hat z &= C_1x + D_{11}\hat d + D_{12}u\end{aligned}$$

where

$$x = \bar x_2, \qquad u = \tau_2$$

$$\hat d = [\tilde d_1\quad \tilde d_2\quad \tilde d_3]^T, \qquad \hat z = \left[\tilde z_1\quad \ddot\theta_2\quad \int h_2\quad u\right]^T$$

$$\hat z_1 = \theta_2 + w_{\alpha_2}\alpha_2 + w_{\dot\alpha_2}\dot\alpha_2 + w_{\beta_2}\beta_2 + w_{\dot\beta_2}\dot\beta_2$$

$$A = \begin{bmatrix}0 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\ -3k_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0\\ 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1\\ 1 & 0 & 0 & 0 & 0 & 0 & -4 & 0\end{bmatrix}$$

Table 1 Closed-loop eigenvalues in units of orbital rate

	Momentum/ attitude	Disturbance filter
Pitch	$-1.27, -2.43$	$-0.30 \pm 1.40j$
	$-0.23 \pm 0.25j$	$-1.40 \pm 2.65j$
Roll/Yaw	$-0.65, -1.77$	$-0.10 \pm 1.00j$
	$-0.44 \pm 0.05j$	$-0.30 \pm 1.09j$
	$-0.10 \pm 1.10j$	$-0.17 \pm 2.00j$
	$-1.26 \pm 0.97j$	$-0.72 \pm 2.38j$

Table 2 Comparison of fastest closed-loop poles

Controller	Fastest closed-loop poles
Ref. 1	−1.50
Ref. 5	−8.29
Ref. 6	−4.77
Ref. 7	−5.43
New design	−2.43

$$B_1 = \begin{bmatrix} 0 & 0 & 0 \\ 3k_{12} & 1 & -3k_{32} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} W_{\hat{d}}, \qquad B_2 = \begin{bmatrix} 0 \\ -1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & w_{\alpha_2} & w_{\dot{\alpha}_2} & w_{\beta_2} & w_{\dot{\beta}_2} \\ -3k_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$D_{11} = \begin{bmatrix} 0 & 0 & 0 \\ 3k_{12} & 1 & -3k_{32} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} W_{\hat{d}}, \qquad D_{12} = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

and $W_{\hat{d}}$ denotes a diagonal weighting matrix for $\hat{d}$; and $(w_{\alpha_2}, w_{\dot{\alpha}_2}, w_{\beta_2}, w_{\dot{\beta}_2})$ are weighting factors for $(\alpha_2, \dot{\alpha}_2, \beta_2, \dot{\beta}_2)$. Note that $\int h_2$ is included as a controlled output variable for the purpose of satisfying assumption (c) in Theorem 3.

The selection of a proper γ and various weighting factors requires a trial-and-error iteration for proper tradeoffs between performance and robustness. Following the procedure presented in Sec. II, a robust H_∞ full-state feedback controller for the pitch axis has been obtained for

$$\gamma = 1, \qquad W_{\hat{d}} = \mathrm{diag}\{0.0002,\ 0.0002,\ 0.0002\}$$

$$w_{\alpha_2} = 20, \qquad w_{\dot{\alpha}_2} = 5, \qquad w_{\beta_2} = 5, \qquad w_{\dot{\beta}_2} = 30$$

The nominal closed-loop eigenvalues of the pitch axis with this controller are listed in Table 1. These closed-loop poles are comparable to those of previous designs,[1,5–7] as can be seen in Table 2 in which the various previous designs are compared in terms of their fastest closed-loop poles. As discussed in Ref. 5, a typical H_∞ control design often achieves the desired robustness by having a high bandwidth controller. As will be discussed in Sec. VI, this new design with the consideration of the nonzero D_{11} term, however, has a remarkable stability robustness margin with nearly the same bandwidth as the conventional linear quadratic regulator (LQR) design. Detailed discussion of this new design in terms of its stability robustness with respect to the moments-of-inertia variations will be given in Sec. VI.

V. Roll/Yaw Control Design

The nondimensionalized model for roll/yaw control design is written as

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & J_1/J_1^0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & J_3/J_3^0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\omega}_1 \\ \dot{\theta}_3 \\ \dot{\omega}_3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 1 & 0 \\ -3(J_2 - J_3)/J_1^0 & 0 & 0 & -(J_2 - J_3)/J_1^0 \\ -1 & 0 & 0 & 1 \\ 0 & (J_2 - J_1)/J_3^0 & 0 & 0 \end{bmatrix}$$

$$\times \begin{bmatrix} \theta_1 \\ \omega_1 \\ \theta_3 \\ \omega_3 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ -1 & 0 \\ 0 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \tau_1 \\ \tau_3 \end{bmatrix} \tag{33}$$

Similar to the pitch control design, the external disturbance d is not included in the preceding equation since it is accommodated by the disturbance rejection filters.

Substituting Eq. (32) into Eq. (33) and decomposing the perturbation matrices, we obtain

$$\Delta E = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \delta_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \delta_3 \end{bmatrix} = M_E \mathcal{E}_E N_E$$

$$\Delta F = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -3k_{21}\delta_2 + 3k_{31}\delta_3 & 0 & 0 & -k_{21}\delta_2 + k_{31}\delta_3 \\ 0 & 0 & 0 & 0 \\ 0 & k_{23}\delta_2 - k_{13}\delta_1 & 0 & 0 \end{bmatrix}$$

$$= M_F \mathcal{E}_F N_F$$

and

$$M_E = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, \qquad M_F = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -k_{21} & 0 & k_{31} \\ 0 & 0 & 0 & 0 \\ -k_{13} & 0 & k_{23} & 0 \end{bmatrix}$$

$$N_E = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad N_F = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 3 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 3 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathcal{E}_E = \mathrm{diag}\{\delta_1, \delta_3\}, \qquad \mathcal{E}_F = \mathrm{diag}\{\delta_1, \delta_2, \delta_2, \delta_3\}$$

where

$$k_1 = \frac{J_2^0 - J_3^0}{J_1^0}, \qquad k_3 = \frac{J_2^0 - J_1^0}{J_3^0}, \qquad k_{ij} = \frac{J_i^0}{J_j^0}$$

Defining the following new variables

$$\tilde{z} = [\dot{\omega}_1 \quad \omega_1 \quad (\omega_3 + 3\theta_1) \quad \omega_1 \quad (\omega_3 + 3\theta_1) \quad \dot{\omega}_3]^T$$

$$= [\tilde{z}_1 \quad \tilde{z}_2 \quad \tilde{z}_3 \quad \tilde{z}_4 \quad \tilde{z}_5 \quad \tilde{z}_6]^T$$

$$\tilde{d} = [\tilde{d}_1 \quad \tilde{d}_2 \quad \tilde{d}_3 \quad \tilde{d}_4 \quad \tilde{d}_5 \quad \tilde{d}_6]^T = -\mathcal{E}\tilde{z}$$

where

$$\mathcal{E} = \mathrm{diag}\{\delta_1,\ \delta_1,\ \delta_2,\ \delta_2,\ \delta_3,\ \delta_3\}$$

we obtain a state-space model of the roll/yaw dynamics, augmented by the fictitious internal feedback loop, as follows:

$$\dot{x} = Ax + B_1\tilde{d} + B_2 u \tag{34a}$$

$$\tilde{z} = C_1 x + D_{11}\tilde{d} + D_{12}u \tag{34b}$$

where

$$x = [\theta_1 \quad \omega_1 \quad \theta_3 \quad \omega_3]^T, \qquad u = [\tau_1 \quad \tau_3]^T$$

$$A = \begin{bmatrix} 0 & 1 & 1 & 0 \\ -3k_1 & 0 & 0 & -k_1 \\ -1 & 0 & 0 & 1 \\ 0 & k_3 & 0 & 0 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} 0 & 0 \\ -1 & 0 \\ 0 & 0 \\ 0 & -1 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & k_{21} & 0 & -k_{31} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & k_{13} & 0 & -k_{23} & 0 & 1 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} -3k_1 & 0 & 0 & -k_1 \\ 0 & 1 & 0 & 0 \\ 3 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 3 & 0 & 0 & 1 \\ 0 & k_3 & 0 & 0 \end{bmatrix}, \qquad D_{12} = \begin{bmatrix} -1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -1 \end{bmatrix}$$

$$D_{11} = \begin{bmatrix} 1 & 0 & k_{21} & 0 & -k_{31} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & k_{13} & 0 & -k_{23} & 0 & 1 \end{bmatrix}$$

Augmenting the perturbed space station equations in Eq. (34) with CMG momentum and the periodic-disturbance filters, we obtain a state-space model for robust H_∞ controller design with state, control input, and controlled output vectors chosen as

$$x = \begin{bmatrix} \bar{x}_1 \\ \bar{x}_3 \end{bmatrix}, \qquad u = \begin{bmatrix} \tau_1 \\ \tau_3 \end{bmatrix}$$

$$\hat{z} = \begin{bmatrix} \tilde{z}_1 & \tilde{z}_2 & \tilde{z}_3 & \tilde{z}_6 & \hat{z}_5 & \int h_1 & \hat{z}_7 & \int h_3 & \tau_1 & \tau_3 \end{bmatrix}^T$$

where

$$\hat{z}_5 = h_1 + w_{\alpha_1}\alpha_1 + w_{\dot{\alpha}_1}\dot{\alpha}_1 + w_{\beta_1}\beta_1 + w_{\dot{\beta}_1}\dot{\beta}_1$$

$$\hat{z}_7 = \theta_3 + w_{\alpha_3}\alpha_3 + w_{\dot{\alpha}_3}\dot{\alpha}_3 + w_{\beta_3}\beta_3 + w_{\dot{\beta}_3}\dot{\beta}_3$$

Similar to the pitch control design, $\int h_1$ and $\int h_3$ are included as controlled output variables for the purpose of satisfying assumption (c) in Theorem 3.

We select $\gamma = 1$ and other weighting factors as follows:

$$W_{\hat{d}} = \text{diag}\{0.01,\ 0.01,\ 0.01,\ 0.05,\ 0.01,\ 0.01\}$$

$$w_{\alpha_1} = 0, \qquad w_{\dot{\alpha}_1} = 0.3, \qquad w_{\beta_1} = 0, \qquad w_{\dot{\beta}_1} = 1$$

$$w_{\alpha_3} = 0.9, \qquad w_{\dot{\alpha}_3} = 1, \qquad w_{\beta_3} = 0.1, \qquad w_{\dot{\beta}_3} = 9$$

Similar to the pitch-axis design, a roll/yaw state feedback gain matrix has been found using the procedure presented in Sec. II. The nominal roll/yaw closed-loop eigenvalues with this controller are listed in Table 1, which are comparable with those of other previous designs. As will be discussed in the next section, this new roll/yaw controller has significant improvement in stability margins over the standard LQR design. Detailed discussion of this new design in terms of its stability robustness will be given in the following section.

VI. Robustness Analysis

In this section, we discuss the effects of the moments-of-inertia variations on the closed-loop stability of the space station with the controllers designed in the preceding sections. The significance of employing an uncertain plant model with $D_{11} \neq 0$ is emphasized.

Directional Inertia Variations

From the definition of the moments of inertia, we have the following physical constraints for possible inertia variations in the three-dimensional parameter space (J_1, J_2, J_3):

$$J_1 + J_2 > J_3, \qquad J_1 + J_3 > J_2, \qquad J_2 + J_3 > J_1 \tag{35}$$

A control designer may unknowingly consider inertia variations that result in inertia values that violate these physical constraints. In such a case, stability of the closed-loop control system is being tested for physically impossible inertia values.

When gravity-gradient torque is used in the control of an orbiting spacecraft, additional inertia constraints are also required as follows:

$$J_1 \neq J_2, \qquad J_1 \neq J_3, \qquad J_2 \neq J_3 \tag{36}$$

As can be noticed in Eq. (19), roll-axis gravity-gradient and gyroscopic coupling torques become zero if $J_2 = J_3$, pitch-axis gravity-gradient torque becomes zero if $J_1 = J_3$, and yaw-axis gyroscopic coupling torque becomes zero if $J_1 = J_2$. For a particular space station configuration considered in this paper, the physical constraints given by Eqs. (35) and (36) can be combined into the following constraints:

$$J_1 + J_2 > J_3, \qquad J_1 \neq J_2, \qquad J_1 \neq J_3 \tag{37}$$

Table 3 summarizes the physical inertia bounds along the various directions of inertia variation for the particular configuration with $J_1 = 50.28 \times 10^6$, $J_2 = 10.80 \times 10^6$, and $J_3 = 58.57 \times 10^6$ slug-ft^2.[5] In Table 3, δ represents the amounts of directional parameter variations with respect to the nominal inertias J_1^0, J_2^0, and J_3^0. It is evident in Table 3 that there exist physical bounds for δ due to the inherent physical properties of the gravity-gradient stabilization and the moments of inertia itself. In particular, the Δ_1-inertia variation is physically caused by the translational motion of the payload along the pitch axis. The robust control design in Ref. 5 was primarily concerned with the Δ_1- and Δ_2-inertia variations.

In Tables 4 and 5, stability margins of the new design are compared with those of the previous designs. Compared with a standard LQR controller of Ref. 1 and an H_∞ controller of Ref. 5, the new design has better stability margins for all Δ_i-inertia variations. A significant margin of 77% for the Δ_2-inertia variation was achieved for the pitch axis, compared with the 34% margin of the LQR design. A significant margin of 77% for the Δ_2-inertia variation was also achieved for the roll/yaw axes, compared with the 43% margin of the standard LQR design.

Table 3 Physical bounds for inertia variations

Variation type $\Delta_i = [\Delta J_1\ \Delta J_2\ \Delta J_3]$	Lower bound $\underline{\delta}$, %	Upper bound $\bar{\delta}$, %
$\Delta_1 = \delta[J_1^0 \quad 0 \quad J_1^0]$	-78.5^{a}	∞
$\Delta_2 = \delta[J_1^0 \quad J_2^0 \quad J_3^0]$	-100.0^{b}	∞
$\Delta_3 = \delta[J_1^0 \quad 0 \quad -J_3^0]$	-2.3^{b}	$+7.6^{c}$
$\Delta_4 = \delta[J_1^0 \quad -J_2^0 \quad 0]$	-6.4^{b}	$+16.4^{c}$
$\Delta_5 = \delta[J_1^0 \quad J_2^0 \quad -J_3^0]$	-2.1^{b}	$+7.6^{c}$

[a]Due to roll/yaw open-loop characteristic.
[b]Due to triangle inequalities for the moments of inertia.
[c]Due to pitch open-loop characteristic.

Table 4 Pitch-axis stability robustness comparison

	Ref. 1		Ref. 5		New design	
%	$\underline{\delta}$	$\bar{\delta}$	$\underline{\delta}$	$\bar{\delta}$	$\underline{\delta}$	$\bar{\delta}$
Δ_1	−99	∞	−99	∞	−99	∞
Δ_2	−89	34	−99	70	−99	77
Δ_3	−17	7.6	−27	7.6	−40	7.6
Δ_4	−19	16	−40	16	−45	16
Δ_5	−30	7.6	−31	7.6	−44	7.6

Table 5 Roll/yaw stability robustness comparison

	Ref. 1		Ref. 5		New design	
%	$\underline{\delta}$	$\bar{\delta}$	$\underline{\delta}$	$\bar{\delta}$	$\underline{\delta}$	$\bar{\delta}$
Δ_1	−78	44	−78	73	−78	76
Δ_2	−99	43	−99	71	−98	77
Δ_3	−61	80	−58	77	−79	79
Δ_4	−64	64	−64	99	−64	99
Δ_5	−51	68	−49	66	−74	67

Compared with the LQR design in Ref. 1, the overall stability robustness with respect to inertia variations has been significantly improved while meeting the nominal performance requirements. We also notice in Table 4 that the new pitch-axis control design slightly improved the stability margin over the design in Ref. 5. On the contrary to the result in Ref. 5 with the fastest closed-loop pole at $-8.38n$, this new design with the fastest closed-loop pole at $-2.43n$ has achieved significant improvement of stability margin with respect to inertia variations. That is, the proposed method with the consideration of the nonzero D_{11} term has resulted in a remarkable stability robustness margin with nearly the same bandwidth as the conventional LQR design.

Real Parameter Margins

A "hypercube" in the space of the plant parameters, centered at a nominal point, is often used as a stability robustness measure in the presence of parametric uncertainty.[16] To determine the largest hypercube that will fit within the existing, but unknown, region of closed-loop stability in the plant's parameter space, consider the open-loop characteristic equations for pitch and roll/yaw axes given by the following:

Pitch axis:

$$J_2 s^2 + 3(J_1 - J_3) = 0 \tag{38}$$

Roll/yaw axes:

$$J_1 J_3 s^4 + \left(-J_1 J_2 + 2J_1 J_3 + J_2^2 + 2J_2 J_3 - 3J_3^2\right)s^2 + 4\left(-J_1 J_2 + J_1 J_3 + J_2^2 - J_2 J_3\right) = 0 \tag{39}$$

Equation (38) represents a characteristic equation of a conservative plant with multilinearly uncertain parameters.[16] For such a system, the ∞-norm real parameter margin of the closed-loop system can be simply found by checking for instability in the corner directions of the parameter space hypercube, at a finite number of critical frequencies. The ∞-norm parameter margin for the pitch axis with the controller designed in Sec. IV can be found as 0.076 at a critical corner with

$$(\delta_1,\ \delta_2,\ \delta_3) = (0.076,\ 0.076,\ -0.076)$$

This corresponds to a critical corner with zero pitch-axis gravity-gradient control torque. However, notice that this largest stable hypercube includes physically impossible inertia variations and that it is too conservative since the controller has 77% margin for the most physically possible Δ_2-inertia variation. The important point is that the control designer should consider only inertia variations that do not violate the physical constraints.

The roll/yaw characteristic Eq. (39) represents a plant that is conservative but not multilinear with respect to the uncertain parameters (J_1, J_2, J_3). Since there is no guarantee that the roll/yaw closed-loop instability with respect to inertia variations occurs at one of the corners of the parameter space hypercube, the real parameter margin computation is not as simple as the case of pitch axis. The inertia boundaries may be computed by iteratively varying the inertia values until a physical bound is reached or the closed-loop system becomes unstable.

VII. Conclusions

A robust control synthesis technique for uncertain dynamical systems subject to inertia matrix variations has been presented. This technique was applied to the full-state feedback control design problem of the space station, resulting in remarkable stability margins with respect to the moments-of-inertia uncertainty over the conventional LQR designs as well as the previous H_∞ design. The uncertain plant model with nonzero D_{11} term was shown to be a proper way of accommodating the uncertain inertia variations in the design of a parameter-insensitive controller.

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